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ON PHYSIOLOGICAL, AS DISTINGUISHED FROM GEOMETRICAL, SPACE.[1]

THE sensible space of our immediate perception, which we find ready at hand on awakening to full consciousness, is considerably different from geometrical space. Our geometrical concepts have been reached for the most part by deliberate experience. The space of the Euclidean geometry is everywhere and in all directions constituted alike; it is unbounded and it is infinite in extent. On the other hand, the space of sight, or "visual space" as it has been termed by Johannes Müller and Hering, is found to be neither constituted everywhere and in all directions alike, nor infinite in extent, nor unbounded. The facts relating to the vision of forms, which I have discussed in another place,[2] show that entirely different feelings correspond to "upness" and "downness," as well as to "nearness" and "farness." "Rightness" and "leftness" also rest on different feelings, although in this case the similarity, owing to considerations of physiological symmetry,[2] is greater. The unlikeness of different directions finds its expression in the phenomena of physiological similarity. The apparent augmentation of the stones at the entrance to a tunnel as we rapidly approach it in a railway train, the shrinking of the same objects on the train's

[1] This article, which rests on researches begun almost forty years ago, are at variance with the views on sensible space that Prof. H. Poincaré advanced in the first part of his paper, "On the Foundations of Geometry," *The Monist*, Vol. IX. page 1 et seq. But my disagreement on this point has no bearing on the subsequent discussions of Professor Poincaré.—*E. Mach.* (Translated by T. J. McCormack.)

[2] *Analysis of the Sensations*, 1886. English trans. Chicago, 1897, p. 49 et seq.

emerging from the tunnel, are exceptionally distinct cases only of the fact of daily experience that objects in visual space cannot be moved about without suffering expansion and contraction,—so that the space of vision resembles in this respect more the space of the metageometricians than it does the space of Euclid.

Even familiar objects *at rest* exhibit the same peculiarities. A long cylindrical glass vessel tipped over the face, a walking-stick laid endwise against one of the eyebrows, appear strikingly conical in shape. The space of our vision is not only bounded, but at times it appears to have even very narrow boundaries. It has been shown by an experiment of Plateau that an after-image no longer suffers appreciable diminution when projected upon a surface the distance of which from the eye exceeds thirty meters. All ingenuous people, depending on direct perception, like the astronomers of antiquity, see the heavens approximately as a sphere, finite in extent. In fact, the oblateness of the celestial vault vertically,—a phenomenon with which even Ptolemy was acquainted, and which Euler has discussed in modern times,—is proof that our visual space is of unequal extent even in different directions. Zoth appears to have found a physiological explanation of this fact, closely related to the conjecture of Ptolemy. The narrow boundaries of space follow, indeed, directly from the possibility of panoramic painting.

Visual space has but few properties in common with geometric space. Both spaces are threefold manifoldnesses. To every point of geometric space, A, B, C, D, corresponds a point A', B', C', D', of visual space. If C lies between B and D, then also will C lie between B' and D'. It is also permissible to say that to a continuous motion of a point in geometric space there corresponds a continuous motion of a co-ordinate point in visual space. I have remarked elsewhere that this continuity, which is merely a convenient fiction, need not in the case of either space be an actual continuity. As every system of sensations, so also the system of space-sensations, is finite,—a fact which cannot astonish us. An endless series of sensational qualities or intensities is psychologically inconceivable. The other properties of visual space also are adapted

to biological conditions. The biological needs would not be satisfied with the pure relations of geometric space. "Rightness," "leftness," "aboveness," "belowness," "nearness," and "farness," must be distinguished by a sensational quality. The locality of an object, and not merely its relation to other localities, must be known, if an animal is to profit by such knowledge. It is also advantageous that the sensational indices of visual objects which are near by and consequently more important biologically, are sharply graduated; whereas, economy is practiced with the limited stock of indices at hand in the case of remote and less important objects.

We shall now develop a simple general consideration, which is again essentially of a teleological nature. Let several distinct spots on the skin of a frog be successively irritated by drops of acid; the frog will respond to each of the several irritations with a specific movement of defense corresponding to the spot irritated. Qualitatively like stimuli affecting different elementary organs and entering by different paths give rise to processes which are propagated back to the environment of the animal again by different organs along different paths. As self-observation shows, we recognise the sameness of the irritational quality of a burn at whatever sensitive spot it may occur, but distinguish the spots irritated; and our conscious or unconscious movement for protection is executed accordingly. The same holds true for itching, tickling, pressure on the skin, etc. We may be permitted to assume, accordingly, that in all these cases there is resident in the sensation, which qualitatively is the same, some differentiating constituent which is due to the specific character of the elementary organ or spot irritated, or, as Hering would say, to the locality of the attention. Conditions resembling those which hold for the skin doubtless also obtain for the extended surface of any sensory organ; although, as in the case of the retina, the facts are here somewhat more complicated. Instead of movements for protection or flight may appear also, conformably with the quality of the irritation, movements[1] of

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[1] I accept, it will be seen, in a somewhat modified and extended form, the opinion advanced by Wlassak. Cf. his beautiful report, "Ueber die statischen Functionen des Ohrlabyrinths," *Vierteljahrssch. f. w. Philos*, XVII. 1 s. 29.

attack, the form of which is also determined by the spot irritated. The snapping reflex of the frog which is produced optically, and the picking of young chicks, may serve as examples. *The mutual biological adaptation of large groups of connected elementary organs is thus very distinctly expressed in the perception of space.*

This natural and ingenuous view leads directly to the theory advanced by Prof. William James, according to which *every* sensation is in part spatial in character; a distinct locality, determined by the element irritated, being its invariable accompaniment. Since generally a plurality of elements enters into play, *voluminousness* would also have to be ascribed to sensations. In support of his hypothesis James frequently refers to Hering. This conception is, in fact, almost universally accepted for optical, tactile, and organic sensations. Many years ago, I myself characterised the relationship of tones of different pitch as spatial, or rather as analogous to spatial; and I believe that the casual remark of Hering, that deep tones occupy a greater volume than high tones, is quite apposite.[1] The highest audible notes of Koenig's rods give as a fact the impression of a needle-thrust, while deep tones appear to fill the entire head. The possibility of localising sources of sound, although not absolute, also points to a relation between sensations of sound and space. In the first place, we clearly distinguish, in the case of high tones at least, whether the right or the left ear is more strongly affected. And although the parallel between binocular vision and binaural audition, which Steinhauser[2] assumes, may possibly not extend very far, there exists, nevertheless, a certain analogy; and the localising of sources of sound is effected preferentially by the agency of high tones[3] (of small volume and more sharply distinguished locality).

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[1] I am unable to give the reference for this remark definitely; it was therefore doubtless made to me orally. Germs of a like view, as well as suggestions toward the modern physical theories of audition, are to be found even in Johannes Müller (*Zur vergleich. Physiolog. des Gesichtssinnes*, Leipsic, 1826, p. 455 et seq.).

[2] Steinhauser, *Ueber binaureales Hören*. Vienna. 1877.

[3] *Ueber die Funktion der Ohrmuschel*. Tröltsch, *Archiv für Ohrenheilkunde*, N. F., Band 3, S. 72.

The physiological spaces of different senses embrace in general physical domains which are only in part coincident. Almost the entire surface of the skin is accessible to the sense of touch, but only a part of it is visible. On the other hand, the sense of sight in general extends very much farther physically. We cannot see our internal organs, which, like the elementary organs of sense, we feel as existing in space and invest with locality only when their equilibrium is disturbed; and these same organs fall only partly within range of the sense of touch. Similarly, the determination of position in space by means of the ear is far more uncertain and is restricted to a much more limited field than that by the eye. Yet, loosely connected as the different space-sensations of the different senses may originally have been, they have still entered into connexion through association, and that system which has the greater practical importance at the time being is prepared to take the place of the other (James). The space-sensations of the different senses are undoubtedly related, but they are certainly not identical. It is of little consequence whether all these sensations be termed space-sensations or whether *one* species only be invested with this name and the others be conceived as analogues of them.

If sensation generally, inclusive of sensation of space, be conceived not as an isolated phenomenon, but in its biological function, in its biological relationship, the entire subject will be rendered more intelligible. As soon as an organ or system of organs is irritated, the appropriate movements are induced as reflexes. If in complicated conditions of life these movements be found to be evoked spontaneously in response to a part only of the original irritation, in response to some slight impulse, in response to a memory, then we are obliged to assume that traces corresponding to the character of the irritation as well as the irritated organs must be left behind in the memory. It is intelligible thus that every sensory field has its own memory and its own spatial order.

The physiological spaces are multiple manifoldnesses of sensation. The wealth of the manifoldness must answer to the wealth of the elements irritated. The nearer elements of the same kind lie together, the more nearly they are akin embryologically, and the more

nearly alike are the space-sensations which they produce. If A and B be two elementary organs, it is permissible to assume that the space-sensation produced by each of them is composed of two constituent parts, a and b, of which the one, a, diminishes the more, and the other, b, increases the more, the farther B is removed from A, or, the more the ontogenetic relationship of B to A decreases. The elements situated in the series AB present a continuously graduated onefold manifoldness of sensation. The multiplicity of the spatial manifoldness must be determined in each case by a special investigation; for the skin, which is a closed surface, a twofold manifoldness would suffice, although a multiple manifoldness is not excluded, and is, by reason of the varying importance of different parts of the skin, even very probable.

It may be said that sensible space consists of a *system of feelings evoked by the sensory organs*, which, while they would not exist without the sense-impressions arising from these organs, yet when aroused by the latter *constitute a sort of scale in which our sense-impressions are registered.* Although every single feeling due to a sensory organ (feeling of space) is registered according to its specific character between those next related to it, a plurality of excited organs is nevertheless very advantageous for distinctness of localisation, for the reason that the contrasts between the feelings of locality are enlivened in this way. Visual space, therefore, which ordinarily is well filled with objects, also affords the best means of localisation. Localisation becomes at once uncertain and fluctuant for a single bright spot on a dark background. (S. Exner.)

It may be assumed that the system of space-sensations is in the main very similar, though unequally developed, in all animals which, like man, have three cardinal directions distinctly marked on their bodies. Above and below, the bodies of such animals are unlike, as they are also in front and behind and to the right and to the left. To the right and the left, these animals are apparently alike, but their geometrical and mechanical symmetry, which subserves purposes of rapid locomotion, should not deceive us with regard to their anatomical and physiological asymmetry. Though the latter may appear slight, it is yet distinctly marked in the fact

that species very closely allied to symmetrical animals sometimes assume strikingly unsymmetrical forms. The asymmetry of the plaice (flatfish) is a familiar instance, while the externally symmetric form of the slug forms an instructive contrast to the unsymmetric shapes of some of its nearer relatives. This trinity of conspicuously marked cardinal directions might indeed be regarded as the physiological basis for our familiarity with the three dimensions of geometric space.

Visual space forms the clearest, precisest, and broadest system of space-sensations, but biologically tactual space is perhaps more important. Irritations of the skin are spatially registered from the very outset; they disengage the corresponding protective movement; the disengaged movement then again induces sensations in the extended or contracted skin, in the joints, in the muscles, etc., which are associated with sensations of space. The first localisations in tactual space are presumably effected on the body itself, the palm of the hand being carried, for example, over the surface of the thigh, which also is sensitive to impressions of space. In this manner, experiences in the field of tactual space are gathered. But the attempt which is frequently made of deriving tactual space psychologically from such experiences, by aid of the concept of time and on the assumption of spaceless sensations, is an altogether hopeless undertaking.

It is my opinion that the space of touch and the space of vision may be conceived after quite the same manner. This can be done, so far as I can infer from what has already been attempted in this direction, only by transferring Hering's view of visual space to tactual space. This also accords best with general biological considerations. A newly-hatched chick notices a small object, looks toward it, and immediately pecks at it. A certain area in the central organ is excited by the irritation, and the looking movement of the muscles of the eye, as well as the picking movements of the head and neck, are forthwith automatically disengaged thereby. The excitation of the above-mentioned area of the central organ, which on the one hand is determined by the geometric locality of the physical irritation, is on the other hand the basis of the space-

sensation. The disengaged muscular movements themselves become a source of sensations in greatly varying degree. Whereas the sensations attending the movements of the eyes, in the case of man at least, usually disappear almost altogether, the movements of the muscles made in the performance of work leave behind them a powerful impression. The behavior of the chick is quite similar to that of an infant which spies a shining object and snatches at it. It will scarcely be doubted that in addition to optical irritations other irritations, acoustic, thermal, and gustatory in character, are also able to evoke movements of prehension or defense, especially so in the case of blind people, and that to the same movements, the same irritated parts of the central organ, and therefore also the same sensations of space, will correspond. The irritations affecting blind people are, as a general thing, merely limited to a more restricted sphere and less sharply determined as to locality. The system of spatial sensations of such people may at first be more meagre and more obscure; consider, for instance, the situation of a blind person endeavoring to protect himself from a wasp buzzing around his head. Yet education can do very much towards perfecting the spatial sense of blind people, as the achievements of the blind geometer Saunderson clearly show. Spatial orientation must notwithstanding have been somewhat difficult for him, as is proved by the construction of his table, which was divided in the simplest manner into quadratic spaces. He was wont to insert pins into the corners and centers of these squares and to connect their heads by threads. His highly original work, however, must by reason of its very simplicity have been particularly easy for beginners to understand; thus he demonstrated the proposition that the volume of a pyramid is equal to one third of the volume of a prism of the same base and height, by dividing a cube into six congruent pyramids, each having a side of the cube for its base and its vertex in the center of the cube.[1]

Tactual space exhibits the same peculiarities of anisotropy and of dissimilarity in the three cardinal directions as visual space,

[1] Diderot, *Lettre sur les aveugles.*

and differs in these peculiarities also from the geometric space of Euclid. On the other hand, optical and tactual space-sensations are at many points in accord. If I stroke with my hand a stationary surface having upon it distinct tangible objects, I shall feel these objects as at rest, just as I should feel visual objects to be when voluntarily causing my eyes to pass over them, although the images themselves actually move across the retina. On the other hand, a moving object appears in motion to the seeing or touching organ either when the latter is at rest or when it is following the object. Physiological symmetry and similarity find the same expression in the two domains, as has been elsewhere shown in detail;[1] but, however intimately allied they may be, the two systems of space-sensations cannot nevertheless be identical. When an object excites me in one case to look at it and in another to grasp it, certainly the portions of the central organ which are affected must be in part different, no matter how nearly contiguous they may be. If both results take place, the domain is naturally larger. For biological reasons, we may expect that the two systems readily coalesce by association, and readily adapt themselves to one another, as is actually the case.

But the province of the phenomena with which we are concerned is not yet exhausted. A chick can look at an object, pick at it, or even be determined by the stimulus presented to run to it, turn towards or around to it. A child that is creeping toward an objective point, and then some day gets up and runs with several steps toward it, acts likewise. We are under the necessity of conceiving these cases, which pass continuously into one another, from some similar point of view. There must be certain parts of the brain which, having been irritated in comparatively simple manner, on the one hand give rise to feelings of space and on the other hand, by their organisation, produce automatic movements which at times may be quite complicated. The stimulus to extensive locomotion and change of orientation not only proceeds from optical excitations, but may also be induced, even in the case of blind

[1] *Analysis of the Sensations*, Eng. trans., p. 50 et seq.

animals, by chemical, thermal, acoustic, and galvanic excitations.[1] In point of fact, we also observe extensive movements of locomotion and orientation in animals that are constitutionally blind (blind worms), as well as in such as are blind by retrogression (moles and cave animals). We may accordingly conceive sensations of space as determined in a perfectly analogous manner both in animals with and in animals without sight.

A person watching a millipede creeping uniformly along is irresistibly impressed with the idea that there proceeds from some organ of the animal a uniform stream of stimulation which is answered by the motor organs of its successive segments with rhythmic automatic movements. Owing to the difference of phase of the hind as compared with the fore segments, there is produced a longitudinal wave which we see propagated through the legs of the animal with mechanical regularity. Analogous phenomena cannot be wanting in the higher animals, and as a matter of fact do exist there. We have an analogous case during active or passive rotation about the vertical axis, when the irritation induced in the labyrinth disengages the well-known nystagmic movements of the eyes. The organism adapts itself so perfectly to certain regular alterations of excitations that on the cessation of these alterations under certain circumstances negative after-images are produced. I have but to recall to the reader's mind the experiment of Plateau and Oppel with the expanding spiral, which when brought to rest appears to shrink, and the corresponding results which Dvorák produced by alterations of the intensity of light. Phenomena of this kind led me long ago to the assumption that there corresponded to an alteration of the stimulus u with the time t, to a rate of alteration, $\frac{du}{dt}$, a special process which under certain circumstances might be felt and which is of course associated with some definite organ. Thus, rate of motion, within the limits within which the perceiving organ can adapt itself, is felt directly; it is therefore not only an abstract idea, as is the speed of the hand of a clock or of a projectile, but it is also a specific sensation, and furnished the

[1] Loeb, *Vergleichende Gehirnphysiologie*, Leipzig, 1899, page 108 et seq.

original impulse to the formation of the idea. Thus, a person feels in the case of a line not only a succession of points varying in position, but also the direction and the curvature of the line. If the intensity of illumination of a surface is given by $u = f(x, y)$, then not only u but also $\frac{du}{dx}, \frac{du}{dy}$, and $\frac{d^2u}{dx^2}, \frac{d^2u}{dy^2}$, find their expression in sensation,—a circumstance which points to a complicated relationship between the elementary organs.

If there actually exists, then, as in the millipede, an organ which on simple irritation disengages the complicated movements belonging to a definite kind of locomotion, it will be permissible to regard this simple irritation, provided it is conscious, as *the will* or *the attention* appurtenant to this locomotion and carrying the latter spontaneously with it. At the same time, it will be recognised as a need of the organism that the effect of the locomotion should be felt in a correspondingly simple manner.

For detailed illustration, we will revert once more to the consideration of visual space. The perception of space proceeds from a biological need, and will be best understood in its various details from this point of view. The greater distinctness and the greater nicety of discrimination exercised at a single specific spot of the retina of vertebrate animals is an economic device. By it, the possibility of moving the eye in response to changes of attention is rendered necessary, but at the same time the disturbing effects of *willed* movements of the eyes on the sensations of space induced by objects at rest have to be excluded. Perception of the movement of an image across the retina when the retina is at rest, perception of the movement of an object when the eye is at rest, is a biological necessity. As for the perception of objects at rest in the unfrequent contingency of a movement of the eye due to some occurrence extrinsic to consciousness (external mechanical pressure, or twitching of the muscles), this was unnecessary for the organism. The foregoing requirements are to be harmonised only on the assumption that the displacement of the image on the retina of the eye in voluntary movement is offset as to spatial value by the volitional character of the movement. It follows from this that objects

at rest may be made, while the eye also is at rest, to suffer displacement in visual space by the *tendency to movement* merely, as has been actually shown by experiment.[1] The second offsetting factor is also directly indicated in this experiment. The organism is not obliged, further, in accomplishing its adaptation, to take account of the second contingency mentioned, which arises only under pathological or artificial circumstances. Paradoxical as the conditions here involved may appear, and far removed as we may still be from a *causal* comprehension of them, they are nevertheless easily understood when thus viewed teleologically as a connected whole.

Shut up in a cylindrical cabinet rotating about a vertical axis, we see and feel ourselves rotating, along with the cylindrical wall, in the direction in which the motion takes place. The impression made by this sensation is at first blush highly paradoxical, inasmuch as there exists not a vestige of a reason for our supposing that the rotation is a relative one. It appears as if it would be actually possible for us to have sensations of movement in absolute space,—a conception to which no physical significance can possibly be attached. But physiologically the case easily admits of explanation. An excitation is produced in the labyrinthine canals of the internal ear,[2] and this excitation disengages, independently of consciousness, a reflex rotary movement of the eyes in a direction opposite to that of the motion,[3] by which the retinal images of all objects resting against the body are displaced exactly as if they were rotating in the direction of the motion. Fixing the eyes intentionally upon some such object, the rotation does not, as might be supposed, disappear. The eye's tendency to motion is then exactly counterbalanced by the introduction of a factor extrinsic to consciousness.[4] We have here the case mentioned above, where the eye, held externally at rest, becomes aware of a displacement in the direction of its tendency to motion. But what before ap-

[1] *Analysis of the Sensations.* English Trans. Page 59.

[2] *Bewegungsempfindungen,* 41 et seq. Leipsic, 1875.

[3] Breuer, *Vorläufige Mittheilung im Anzeiger der kk. Gesellschaft der Aerzte in Wien, vom 20. Nov. 1873.*

[4] *Analysis of the Sensations.* English Trans. Page 71.

peared as a paradoxical exception is now a natural result of the adaptation of the organism, by which the animal perceives the motion of its own body when external objects at rest remain stationary. Analogous adaptive results with which even Purkynje was in part acquainted are met with in the domain of the tactile sense.[1]

The eyes of an observer watching the water rushing underneath a bridge are impelled without noticeable effort to follow the motion of the flowing water and to adapt themselves to the same. If the observer will now look at the bridge, he will see both the latter and himself moving in a direction opposite to that of the water. Here again the eye which fixates the bridge must be maintained at rest by a willed motional effort made in opposition to its unconsciously-acquired motional tendency, and it now sees apparent motions to which no real motions correspond. But the same phenomena which appear here paradoxical and singular undoubtedly serve an important function in the case of progressive motion or locomotion. To the property of the visual apparatus just discussed is due the fact that an animal in progressive motion sees itself moving and the stationary objects in its environment at rest.[2] Anomalies of this character, where a body appears to be in motion without moving from the spot which it occupies, where a body contracts without really growing smaller, which we are in the habit of calling illusions on the few rare occasions when we notice them, have accordingly their important normal and common function.

As the process which we term the *will* to turn round or move forwards is of a very simple nature, so also is the result of this will characterised by feelings of a very simple nature. *Fluent* spatial values of certain objects, instead of stable, make their appearance in the domain of the visual as well as of the tactile sense. But even where visual and tactile sensations are as much as possible excluded, unmistakable sensations of motion are produced; for example, a person placed in a darkened room, with closed eyes, on

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[1] Purkynje, *Beiträge zur Kenntniss des Schwindels. Medizin. Jahrbücher des österreichischen Staates*, VI. Wien, 1820. *Versuche über den Schwindel. 10 Bulletin der naturw. section der schles. Gesellschaft.* Breslau, 1825, s. 25.

[2] *Analysis of the Sensations.* English Trans. Pages 63, 64, 71, 72.

a seat affording support to the body on all sides, will be conscious of the slightest progressive or angular acceleration in the movement of his body, no matter how noiselessly and gently the same may be produced.[1] By association, these simple sensations also are translated at once into the motor images of the other senses. Between this initial and terminal link of the process are situated the various sensations of the extremities moved, which ordinarily enter consciousness, however, only when obstructions intervene.

We have now, as I believe, gained a fair insight into the nature of sensations of space. The last-discussed species of sensations of space, which were denominated sensations of movement, are sharply distinguished from those previously investigated by their *uniformity* and *inexhaustibility*. These sensations of movement make their appearance only in animals that are free to move about, whereas animals that are confined to a single spot are restricted to the sensations of space first considered, which we shall designate *primary* sensations of space, as distinguished from *secondary* sensations (of movement). A *stationary* animal possesses necessarily a *bounded* space. Whether that space be symmetrical or unsymmetrical depends upon the conditions of symmetry of its own body. A vertebrate animal confined to a single spot and restricted as to orientation could only construct a bounded space which was dissimilar above and below, before and behind, and accurately speaking also to the right and to the left, and which consequently would present a sort of analogy with the physical properties of a triclinic crystal. If the animal acquired the power of moving freely about, it would obtain in this way in addition an *infinite* physiological space; for the sensations of movement always admit of being *produced anew* when not prevented by accidental external hindrances. Untrammeled orientation, the interchangeability of every orientation with every other, invests physiological space with the property of equality in all directions. Progressive motion and the possibility of orientation in any direction together render space identically constituted at all places and in all directions. Nevertheless, we may remark at this

[1] *Bewegungsempfindungen*, Leipsic, 1875.

juncture that the foregoing result has not been obtained through the operation of physiological factors exclusively, for the reason that orientation with respect to the vertical, or the direction of the acceleration of gravity, is not altogether optional in the case of any animal. Marked disturbances of orientation with respect to the vertical make themselves most strongly felt in the higher vertebrate animals by their physico-physiological results, by which they are restricted as regards both duration and magnitude. Primary space cannot be absolutely supplanted by secondary space, for the reason that it is phylogenetically and ontogenetically older and stronger. If primary space decreases in significance during motion, the sensation of movement in its turn immediately vanishes when the motion ceases, as does every sensation which is not kept alive by reviviscence and contrast. Primary space then again enters upon its rights. It is doubtless unnecessary to remark that physiological space is in no wise concerned with metrical relations.

We have assumed that *physiological* space is an adaptive result of the interaction of the elementary organs, which are constrained to live together and are thus absolutely dependent upon co-operation, without which they could not exist. Of primary and greatest importance to animals are the parts of *their own body* and their relations to one another; outward bodies come into consideration only in so far as they stand in some way in connexion with the parts of the animal body. The conditions here involved are physiological in character,—which does not exclude the fact that every part of the body continues to be a part of the physical world, and so subject to general physical laws, as is most strikingly shown by the phenomena which take place in the labyrinth during locomotion, or by a change of orientation. *Geometric* space embraces only the relations of physical bodies *to one another*, and leaves the animal body in this connexion altogether out of account.

We are aware of but one species of elements of consciousness: sensations. In our perceptions of space we are dependent on sensations. The character of these sensations and the organs that are in operation while they are being felt, are questions that must be left undecided. The view on which the preceding reflections are

based is as follows: The feeling with which an elementary organ is affected when in action, depends partly upon the character (or quality) of the irritation; we will call this part the *sense-impression.* A second part of the feeling, on the other hand, may be conceived as determined by the *individuality* of the organ, being the same for every stimulus and varying only from organ to organ, the degree of variation being inversely proportional to the ontogenetic relationship. This portion of the feeling may be called the *space-sensation.* Space-sensation can accordingly be produced only when there is some irritation of elementary organs; and every time the same organ or the same complexus of organs is irritated, every time the same concatenation of organs is aroused, the space-sensation is the same. We make only the same assumptions here with regard to the elementary organs which we should deem ourselves quite justified in making with respect to isolated individual animals of the same phylogenetic descent but different degrees of affinity.

The prospect is here opened of a phylogenetic and ontogenetic understanding of spatial perception; and after the conditions of the case have been once thoroughly elucidated, a physical and physiological explanation seems possible. I am far from thinking that the explanation here offered is absolutely adequate or exhaustive on all sides; but I am convinced that I have made some approach to the truth by it.

Kant asserted that "one could never picture to oneself that space did not actually exist, although one might quite easily imagine that there were no objects in space." To-day, scarcely any one doubts that sensations of objects and sensations of space enter consciousness only in combination with one another; and that, *vice versa,* they can leave consciousness only in combination with one another. And the same must hold true with regard to the concepts which correspond to these sensations. If for Kant space is not a "concept," but a "pure (mere?) intuition *a priori,*" modern inquirers on the other hand are inclined to regard space as a concept, and furthermore as a concept which has been derived from experience. We cannot intuite our system of space-sensations *per se*: but we may neglect sensations of objects as something

subsidiary; and if we overlook what we have done, the notion may easily arise that we are actually concerned with a pure intuition. If our sensations of space are independent of the quality of the stimuli which serve to produce them, then we may make predications concerning the former independently of external or physical experience. It is the imperishable merit of Kant to have called attention to this point. This basis is unquestionably inadequate to the complete development of a geometry, inasmuch as concepts, and in addition thereto concepts derived from experience, are also requisite to this purpose.

Physiological, and particularly visual, space appears as a distortion of geometrical space when derived from the metrical data of geometrical space. But the properties of continuity and threefold manifoldness are preserved in such a transformation, and all the consequences of these properties may be derived without recourse to physical experience, by our representative powers solely.

Since physiological space, as a system of sensations, is much nearer at hand than the geometric concepts that are based thereon, the properties of physiological space will be found to assert themselves quite frequently in our dealings with geometric space. We distinguish near and remote points in our figures, those at the right from those at the left, those at the top from those at the bottom, entirely by physiological considerations and despite the fact that geometric space is not cognisant of any relation to our body, but only of relations of the points to one another. Among geometric figures, the straight line and the plane are specially marked out by their physiological properties; as they are indeed the first objects of geometrical investigation. Symmetry is also distinctly revealed by its physiological properties, and attracts thus immediately the attention of the geometer. It has doubtless also been efficacious in determining the division of space into right angles. The fact that similitude was investigated previously to other geometric affinities is likewise due to physiological facts. The Cartesian geometry of co-ordinates in a sense liberated geometry from physiological influences, yet vestiges of their thrall still remain in the distinction of positive and negative co-ordinates, according as

these are reckoned to the right or to the left, upward or downward, and so on. This is convenient, but not necessary. A fourth co-ordinate plane, or the determination of a point by its distances from four fundamental points not lying in the same plane, exempts geometric space from the necessity of constantly recurring to physiological space. The necessity of such restrictions as "around to the right" and "around to the left," and the distinction of symmetrical figures by these means would then be eliminated. The historical influences of physiological space on the development of the concepts of geometric space are, of course, not to be eliminated.

Also in other provinces, as in physics, the influence of the properties of physiological space is traceable, and not alone in geometry. Even secondary physiological space is considerably different from Euclidean space, owing to the fact that the distinction between "above" and "below" does not absolutely disappear in the former. Sosikles of Corinth (Herodotus v. 92) asseverated that "sooner should the heavens be beneath the earth and the earth soar in the air above the heavens, than the Spartans lose their freedom, etc." And his assertion, together with the tirades of Lactantius (*De falsa sapientia*, c. 24) and St. Augustine (*De civitate dei*, XVI., 9), against the doctrine of the antipodes, against men hanging with inverted heads and trees growing downward,—considerations which even after centuries touch in us a sympathetic cord,—all had their good physiological grounds. We have, in fact, less reason to be astonished at the narrow-mindedness of these opponents of the doctrine of the antipodes than we have to be filled with admiration for the great powers of abstraction exhibited by Archytas of Tarentum and Aristarchus of Samos.

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